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Topological field theories and geometry of Batalin-Vilkovisky algebras

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Abstract: The algebraic and geometric structures of deformations are analyzed concerning topological field theories of the Schwarz type by means of the Batalin-Vilkovisky formalism. Deformations of the Chern-Simons-BF theory in three dimensions induces the Courant algebroid structure on the target space as a sigma model. Deformations of BF theories in n dimensions are also analyzed. Two-dimensional deformed BF theory induces the Poisson structure and three-dimensional deformed BF theory induces the Courant algebroid structure on the target space as a sigma model. The deformations of BF theories in n dimensions induce the structures of Batalin-Vilkovisky algebras on the target space.

Keywords: Topological Field Theories, Gauge Symmetry, BRST Symmetry.

Contents

1. Introduction

There are two types in topological field theories. The Witten type and the Schwarz type [1]. The topological field theories of the Schwarz type are those which do not depend on the metric. Roughly speaking, they are the Chern-Simons gauge theory and the BF theories.

The author has made deformation of the BF theories in n dimensions and Chern-Simons gauge theory coupled with the BF theory in three dimensions in the previous papers [2, 3, 4], and found new topological field theories with exotic extended gauge symmetries. The way of analyzing those theories is done with the deformation theory of the gauge theory proposed by Barnich and Henneaux [5, 6]. We investigate moduli of deformations by analyzing the BRST cohomology in the framework of Batalin-Vilkovisky (BV) formalism (the antifield BRST formalism) [7].

We consider an action $S_0[\Phi]$ of the fields Φ and its gauge symmetry δ_0. The action S_0 is gauge invariant under δ_0, that is, $\delta_0 S_0 = 0$. The action and the gauge symmetry are deformed perturbatively as

$$\begin{aligned} S &= S_0 + gS_1 + g^2 S_2 + \cdots , \\ \delta\Phi &= \delta_0\Phi + g\delta_1\Phi + g^2\delta_2\Phi + \cdots , \end{aligned} \tag{1.1}$$

where g is a deformation parameter. The consistency of the deformed theory requires that the deformed action is gauge invariant under the deformed gauge symmetry,

$$\delta S = 0\,, \tag{1.2}$$

and the deformed gauge symmetry is closed, that is,

$$[\delta_\epsilon, \delta_{\epsilon'}] = \delta_{[\epsilon,\epsilon']} \tag{1.3}$$

holds on shell where ϵ and ϵ' are gauge parameters. These conditions are realized as the classical master equation, $(S, S) = 0$, in the BV formalism, where $(\cdot,\cdot)$ is the antibracket and S is a BV action. By deformation of the topological field theory, we mean analysis of the freedoms of deformation of the mathematical structures defined by the topological field theory.

The Chern-Simons gauge theory in three dimensions has the following action:

$$S_{CS} = \int_X \left(\frac{k_{ab}}{2} A^a \wedge dA^b + \frac{1}{6} f_{abc} A^a \wedge A^b \wedge A^c \right), \tag{1.4}$$

where X is a three-dimensional manifold, A^a is a one-form gauge field, $k_{ab} = k_{ba}$ is a non degenerate constant and f_{abc} is the structure constant of a Lie algebra. This theory has the following gauge symmetry:

$$\delta A^a = d\epsilon^a + k^{ab} f_{bcd} A^c \epsilon^d\,, \tag{1.5}$$

and the field strength F^a vanishes as is seen from the equation of motion.

The BF theory in n dimensions has the following action:

$$S = (-1)^{n-p} \int_\Sigma B_{n-p-1\;a} {F_{p+1}}^a\,, \tag{1.6}$$

where ${F_{p+1}}^a$ is the field strength for a p-form gauge field A_p, and $B_{n-p-1\;a}$ is an $n-p-1$-form auxiliary field. The equation of motion is also, ${F_{p+1}}^a = 0$.

The deformation of the Chern-Simons gauge theory from the abelian one has been analyzed by Barnich and Henneaux [5], who have obtained only the known non-abelian Chern-Simons gauge theory. Deformation of the BF theory in two dimensions has been analyzed by Izawa [8]. There are also some similar works [9, 10]. We obtain two-dimensional nonlinear gauge theory (the Poisson sigma model) [11, 12] as the deformation of the BF theory in two dimensions. The theory has an extended gauge symmetry. The author has analyzed deformations of the BF theory in three dimensions [2] and extended to those in n dimensions [3]. The theories obtained have some exotic gauge symmetries which are extensions of the usual gauge symmetries of the Lie algebras. The deformation topological field theory of the Schwarz type has also been discussed by Edgren and Sandstrom [13], who have obtained explicit solutions in four and six dimensions. The author has obtained a deformed topological field theory with extended gauge symmetry for the Chern-Simons gauge theory coupled with BF theory in three dimensions [4].

The global version of extended gauge symmetries are trivial on the physical S-matrix since we consider topological theories. But the gauge symmetries of the theories are not necessarily mutually equivalent on a topologically non-trivial manifold, including a manifold with boundaries. A known example is the deformation of the BF theory in two dimensions, which is the nonlinear gauge theory [8]. We consider this as a sigma model on a two-dimensional base space X, mapping to a target space M. This theory defines the Poisson structure on the target space M as a sigma model [12], and therefore is called the Poisson sigma model. If we quantize this theory on a disc, correlation functions of observables on the boundary of the disc coincide with the deformation quantization on the Poisson manifold M [26]. We can derive the associativity condition of the deformation quantization as the Ward-Takahashi identity of the gauge symmetry. This model has also been analyzed in the context of the L_∞-algebra [14, 15, 16] and of the Lie algebroid [17, 18].

We mainly consider such theories as a sigma model on an n-dimensional base space X, mapping to a target space M. We consider the general deformed BF theories on an n-dimensional manifold X. We analyze their geometrical and algebraic structures induced on the target space M. The n-dimensional topological sigma model defines a topological open $(n-1)$-brane [19]. Therefore we can apply analysis of deformed BF theories to the analysis of deformation of the topological open $(n-1)$-brane.

The key structure for analyzing higher-dimensional deformed theories is the BV structure. The antibracket and the BRST charge is geometrically reformulated as a P-structure and a Q-structure [20, 21]. In this paper, we analyze what structures are induced on the target space by a topological sigma model with P-structure and Q-structure.

In section 2, we consider deformation of the Chern-Simons gauge theory coupled with BF theory in three dimensions and discuss that this theory has the Courant algebroid structure. In section 3 , we briefly review deformation of BF theories in n dimensions. In section 4, we analyze the structures of deformed BF theories in two dimensions and in three dimensions. We point out that the theories have the Poisson structure in two dimensions and the Courant algebroid structure in three dimensions. We reformulate these structures in the BV formalism. In section 5, we analyze the deformed BF theories in n dimensions and their Batalin-Vilkovisky algebras.

2. Chern-Simons-BF theory in three dimensions

2.1 Superfield formalism for the Chern-Simons-BF theory

In this subsection, we briefly review the deformation of the Chern-Simons gauge theory coupled with the BF theory in three dimensions discussed in [4].

Let X be a three-dimensional manifold and M be an N-dimensional manifold. E denotes a vector bundle on M. First we consider the abelian Chern-Simons-BF action,

$$S_0 = \int_X \left(\frac{k_{ab}}{2} A^a \wedge dA^b - B_i \wedge d\phi^i \right), \tag{2.1}$$

where ϕ^i is a 0-form scalar field and is a (smooth) map from X to an N-dimensional target space M. A^a and B_i are a 1-form and a 2-form gauge field, respectively, and k_{ab} is a symmetric constant tensor. We assume that k_{ab} is non degenerate and invertible. Indices a, b, c represent those on the fiber E_x ($x \in M$), and i, j, k, represent the indices on the cotangent bundle T^*M. The sign factor -1 in the front of the second term is introduced for convenience.

The author has analyzed all the BRST cohomology by means of the method developed in [5, 6]. We obtain the following deformation of (2.1):

$$S = \int_X \left(\frac{k_{ab}}{2} A^a \wedge dA^b - B_i \wedge d\phi^i + f_{1a}{}^i(\phi) A^a B_i + \frac{1}{6} f_{2abc}(\phi) A^a A^b A^c \right), \tag{2.2}$$

where $f_{1a}{}^i(\phi)$ and $f_{2abc}(\phi)$ satisfy the following identities:

$$\begin{aligned} k^{ab} f_{1a}{}^i(\phi) f_{1b}{}^j(\phi) &= 0\,, \\ \frac{\partial f_{1b}{}^i(\phi)}{\partial \phi^j} f_{1c}{}^j(\phi) - \frac{\partial f_{1c}{}^i(\phi)}{\partial \phi^j} f_{1b}{}^j(\phi) + k^{ef} f_{1e}{}^i(\phi) f_{2fbc}(\phi) &= 0\,, \\ \left(f_{1d}{}^i(\phi) \frac{\partial f_{2abc}(\phi)}{\partial \phi^i} - f_{1c}{}^i(\phi) \frac{\partial f_{2dab}(\phi)}{\partial \phi^i} + f_{1b}{}^i(\phi) \frac{\partial f_{2cda}(\phi)}{\partial \phi^i} - f_{1a}{}^i(\phi) \frac{\partial f_{2bcd}(\phi)}{\partial \phi^i} \right) + & \\ + k^{ef} (f_{2eab}(\phi) f_{2cdf}(\phi) + f_{2eac}(\phi) f_{2dbf}(\phi) + f_{2ead}(\phi) f_{2bcf}(\phi)) &= 0\,. \end{aligned} \tag{2.3}$$

Two additional terms arise in (2.2) as the most general BRST cohomology class of deformation, where we assume that the ghost number in the action is zero [4].

If $f_{1a}{}^i(\phi) = 0$ and if $f_{2abc}(\phi)$ is independent of ϕ, (2.3) reduces to the usual Jacobi identity for the Lie algebra structure constant and we have the usual non-abelian gauge symmetry. In general, however $f_{2abc}(\phi)$ can depend on the fields, and thus the theory is more general than the usual non-abelian gauge symmetry.

The action (2.2) is invariant under the following gauge symmetry:

$$\begin{aligned} \delta A^a &= dc^a + k^{ab} f_{1b}{}^i t_i + k^{ab} f_{2bcd} A^c c^d\,, \\ \delta B_i &= dt_i + \frac{\partial f_{1b}{}^j}{\partial \phi^i} (A^b t_j - c^b B_j) + \frac{1}{2} \frac{\partial f_{2bcd}}{\partial \phi^i} A^b A^c c^d\,, \\ \delta \phi^i &= -f_{1b}{}^i c^b\,, \end{aligned} \tag{2.4}$$

where c^a and t_i are gauge parameters. c^a is a 0-form and t_i is a 1-form. Since this gauge algebra is an open algebra, it is appropriate to analyze the theory in the framework of the Batalin-Vilkovisky formalism (the antifield formalism).

To do this, first we take both c^a and t_i to be Grassmann odd FP ghosts with ghost number 1, and we introduce v_i to be a Grassmann even ghost with ghost number 2. Next we introduce an antifield Φ^+ corresponding to each field Φ. We require $\deg(\Phi) + \deg(\Phi^+) = n$ and $\mathrm{gh}(\Phi) + \mathrm{gh}(\Phi^+) = -1$, where n is the dimension of the base space X ($n = 3$ in this model), $\deg(\Phi)$ and $\deg(\Phi^+)$ are the form degrees of the fields Φ and Φ^+, respectively, and $\mathrm{gh}(\Phi)$ and $\mathrm{gh}(\Phi^+)$ are the ghost numbers of them. For functions, $F(\Phi, \Phi^+)$ and $G(\Phi, \Phi^+)$, of the fields and the antifields, we define the antibracket by

$$(F, G) \equiv \frac{F \overleftarrow{\partial}}{\partial \Phi} \frac{\overrightarrow{\partial} G}{\partial \Phi^+} - \frac{F \overleftarrow{\partial}}{\partial \Phi^+} \frac{\overrightarrow{\partial} G}{\partial \Phi}\,, \tag{2.5}$$

where $\overleftarrow{\partial}/\partial\varphi$ and $\overrightarrow{\partial}/\partial\varphi$ are the right and left differentiations with respect to φ, respectively. For two functionals S and T, their antibracket is defined by

$$(S,T) \equiv \int_X \left(\frac{S\overleftarrow{\partial}}{\partial\Phi}\frac{\overrightarrow{\partial}T}{\partial\Phi^+} - \frac{S\overleftarrow{\partial}}{\partial\Phi^+}\frac{\overrightarrow{\partial}T}{\partial\Phi} \right). \tag{2.6}$$

Properties of the antibracket are summarized in appendix A.

In order to simplify notations and calculations, we employ superfield formalism. A superfield consists of a field, its antifield and their gauge descendant fields. For ϕ^i, A^a and B_i, the corresponding superfields are as follows:

$$\begin{aligned} \boldsymbol{\phi}^i &= \phi^i + B^{+i} + t^{+i} + v^{+i}\,, \\ \mathbf{A}^a &= c^a + A^a + k^{ab}A^+_b + k^{ab}c^+_b\,, \\ \mathbf{B}_i &= v_i + t_i + B_i + \phi^+_i\,. \end{aligned} \tag{2.7}$$

The total degree, defined by $|F| \equiv \mathrm{gh}F + \deg F$, of the component fields belonging to each is common. The total degrees of $\boldsymbol{\phi}^i$, $\mathbf{A}^a$ and $\mathbf{B}_i$ are 0, 1 and 2, respectively. We introduce the notation *dot product* (denoted by $\cdot$) for superfields in order to make sign factors implicit [22]. The definitions and properties of the *dot product* are listed in appendix B. Using the formula (B.7) presented in appendix B, we can rewrite (2.5) as the antibracket on two superfields F and G as follows:

$$(\!(\, F \,,\, G \,)\!) \equiv F\cdot\frac{\overleftarrow{\partial}}{\partial\mathbf{A}^a}\cdot k^{ab}\frac{\overrightarrow{\partial}}{\partial\mathbf{A}^b}\cdot G + F\cdot\frac{\overleftarrow{\partial}}{\partial\boldsymbol{\phi}^i}\cdot\frac{\overrightarrow{\partial}}{\partial\mathbf{B}_i}\cdot G - F\cdot\frac{\overleftarrow{\partial}}{\partial\mathbf{B}_i}\cdot\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^i}\cdot G\,. \tag{2.8}$$

$(\!(\,\cdot\,,\,\cdot\,)\!)$ is graded symmetric and satisfy the graded Leibniz rule and the graded Jacobi identity with respect to the total degree of superfields. That is, (2.8) defines the graded Poisson bracket on superfields. The formulae are listed in appendix B.

The action (2.2) is extended to the BV action in terms of the superfields as follows:

$$S_{BV} = \int_X \left(\frac{k_{ab}}{2}\mathbf{A}^a\cdot d\mathbf{A}^b - \mathbf{B}_i\cdot d\boldsymbol{\phi}^i + f_{1a}{}^i(\boldsymbol{\phi})\cdot\mathbf{A}^a\cdot\mathbf{B}_i + \frac{1}{6}f_{2abc}(\boldsymbol{\phi})\cdot\mathbf{A}^a\cdot\mathbf{A}^b\cdot\mathbf{A}^c \right), \tag{2.9}$$

where the integration over X is understood as that over the 3-form part of the integrand. From now on, we denote this BV action S_{BV} as S. The gauge invariance of the action is equivalent to the following classical master equation:

$$(\!(\, S \,,\, S \,)\!) = 0\,. \tag{2.10}$$

Substituting (2.9) in the condition (2.10), we obtain the identities on the structure functions $f_{1a}{}^b(\boldsymbol{\phi})$ and $f_{2abc}(\boldsymbol{\phi})$ as

$$\begin{aligned} k^{ab}f_{1a}{}^i\cdot f_{1b}{}^j &= 0\,, \\ \left(\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^j}\cdot f_{1b}{}^i\right)\cdot f_{1c}{}^j - \left(\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^j}\cdot f_{1c}{}^i\right)\cdot f_{1b}{}^j + k^{ef}f_{1e}{}^i\cdot f_{2fbc} &= 0\,, \\ \left\{ f_{1d}{}^j\cdot\left(\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^j}\cdot f_{2abc}\right) - f_{1c}{}^j\cdot\left(\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^j}\cdot f_{2dab}\right) + f_{1b}{}^j\cdot\left(\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^j}\cdot f_{2cda}\right) - f_{1a}{}^j\cdot\left(\frac{\overrightarrow{\partial}}{\partial\boldsymbol{\phi}^j}\cdot f_{2bcd}\right)\right\} &+ \\ + k^{ef}(f_{2eab}\cdot f_{2cdf} + f_{2eac}\cdot f_{2dbf} + f_{2ead}\cdot f_{2bcf}) &= 0\,. \end{aligned} \tag{2.11}$$

If we set all the antifields to zero in (2.11), the identity (2.3) is reproduced.

The BRST transformation of each field is calculated from the definition of the BRST transformation $\delta F = (\!(S \, , \, F)\!)$ as

$$
\begin{aligned}
\delta \mathbf{A}^a &= d\mathbf{A}^a + k^{ab} f_{1b}{}^j \cdot \mathbf{B}_j + \frac{1}{2} k^{ab} f_{2bcd} \cdot \mathbf{A}^c \cdot \mathbf{A}^d \,, \\
\delta \mathbf{B}_i &= d\mathbf{B}_i + \left(\frac{\overrightarrow{\partial}}{\partial \phi^i} \cdot f_{1b}{}^j \right) \cdot \mathbf{A}^b \cdot \mathbf{B}_j + \left(\frac{1}{6} \frac{\overrightarrow{\partial}}{\partial \phi^i} \cdot f_{2bcd} \right) \cdot \mathbf{A}^b \cdot \mathbf{A}^c \cdot \mathbf{A}^d \,, \\
\delta \phi^i &= d\phi^i - f_{1b}{}^i \cdot \mathbf{A}^b \,.
\end{aligned}
\tag{2.12}
$$

2.2 Courant algebroid structure of the CSBF theory

We analyze the identities (2.11) on the structure functions f_1 and f_2, which is equivalent to (2.3). The gauge algebra under this theory is the Courant algebroid.

A Courant algebroid is introduced by Courant in order to analyze the Dirac structure as a generalization of the Lie algebra of the vector fields on the vector bundle [23, 24]. A Courant algebroid is a vector bundle $\mathcal{E} \to \mathcal{M}$ and has a non-degenerate symmetric bilinear form $\langle \cdot, \cdot \rangle$ on the bundle, a bilinear operation $\circ$ on $\Gamma(\mathcal{E})$ (the space of sections on $\mathcal{E}$), and a bundle map (called the anchor) $\rho : \mathcal{E} \to T\mathcal{M}$ satisfying the following properties [25]:

$$
e_1 \circ (e_2 \circ e_3) = (e_1 \circ e_2) \circ e_3 + e_2 \circ (e_1 \circ e_3) \,, \tag{2.13}
$$

$$
\rho(e_1 \circ e_2) = [\rho(e_1), \rho(e_2)] \,, \tag{2.14}
$$

$$
e_1 \circ F e_2 = F(e_1 \circ e_2) + (\rho(e_1) F) e_2 \,, \tag{2.15}
$$

$$
e_1 \circ e_2 = \frac{1}{2} \mathcal{D} \langle e_1, e_2 \rangle \,, \tag{2.16}
$$

$$
\rho(e_1) \langle e_2, e_3 \rangle = \langle e_1 \circ e_2, e_3 \rangle + \langle e_2, e_1 \circ e_3 \rangle \,, \tag{2.17}
$$

where e_1, e_2 and e_3 are sections of $\mathcal{E}$, and F is a function on $\mathcal{M}$; $\mathcal{D}$ is a map from functions on $\mathcal{M}$ to $\Gamma(\mathcal{E})$ and is defined by $\langle \mathcal{D} F, e \rangle = \rho(e) F$. Let e^a be a local basis of sections of $\mathcal{E}$. Then (2.17) is written as

$$
\begin{aligned}
e^a \circ (e^b \circ e^c) &= (e^a \circ e^b) \circ e^c + e^b \circ (e^a \circ e^c) \,, \\
\rho(e^a \circ e^b) &= [\rho(e^a), \rho(e^b)] \,, \\
e^a \circ F e^b &= F(e^a \circ e^b) + (\rho(e^a) F) e^b \,, \\
e^a \circ e^b &= \frac{1}{2} \mathcal{D} \langle e^a, e^b \rangle \,, \\
\rho(e^a) \langle e^b, e^c \rangle &= \langle e^a \circ e^b, e^c \rangle + \langle e^b, e^a \circ e^c \rangle \,.
\end{aligned}
\tag{2.18}
$$

We consider the supermanifold $\tilde{X}$ whose grade zero part is a three-dimensional manifold X. In our topological field theory, a base space $\mathcal{M}$ is the space of a (smooth) map from $\tilde{X}$ to a target space M. The fiber of $\mathcal{E}$ is denoted as $\mathcal{V}[1]$, where $\mathcal{V}$ is a vector space and $[p]$ represents the grading shift by p. That is, the parity of the fiber of $\mathcal{E}$ is reversed and the grading of the fiber is 1. The local basis on $\mathcal{V}[1]$ is $e^a = \mathbf{A}^a$. On this space, we define a (graded) symmetric bilinear form $\langle \cdot, \cdot \rangle$, a bilinear operation $\circ$ and a bundle map ρ

as follows:

$$\begin{aligned}
e^a \circ e^b &\equiv \Big(\!\Big((\!(S\,, e^a)\!)\,, e^b \Big)\!\Big), \\
\langle e^a, e^b \rangle &\equiv \Big(\!\Big(e^a\,, e^b \Big)\!\Big), \\
\rho(e^a)F(\phi) &\equiv (\!(e^a\,, (\!(S\,, F(\phi))\!))\!), \\
\mathcal{D}(*) &\equiv (\!(S\,, *)\!).
\end{aligned} \tag{2.19}$$

Then we can easily confirm that the gauge algebra satisfies the conditions (2.13) to (2.17) of the Courant algebroid by the identities (2.11).

Conversely, first we take the local basis $\mathbf{A}^a$ on the fiber (with the reversed parity) of the vector bundle $\mathcal{E}$. We define the graded odd Poisson structure (2.8) on the bundle $\mathcal{E} \oplus T^*[2]\mathcal{M}$, where the grading on the fiber direction of $T^*[2]\mathcal{M}$ is shifted by 2. We can take a Darboux coordinate on the antibracket such that $\langle \mathbf{A}^a, \mathbf{A}^b \rangle = k^{ab}$. We define the operations $\langle \cdot, \cdot \rangle$, $\circ$ and ρ on the basis by

$$\begin{aligned}
\mathbf{A}^a \circ \mathbf{A}^b &= -k^{ac} k^{bd} f_{2cde}(\phi) \cdot \mathbf{A}^e\,, \\
\langle \mathbf{A}^a, \mathbf{A}^b \rangle &= k^{ab}\,, \\
\rho(\mathbf{A}^a)\phi^i &= -f_{1c}{}^i(\phi) k^{ac}\,.
\end{aligned} \tag{2.20}$$

Then the conditions (2.13) to (2.17) of the Courant algebroid are equivalent to the identities (2.11) on f_1 and f_2. The action S is the BRST charge for the Courant algebroid. Since the master eq. (2.10) is equivalent to (2.11), the relations (2.13) to (2.17) is represented by the master equation of the action S.

3. Deformed BF theory in n dimensions

In this section, we briefly review the results of the paper [3], in which the deformed BF theory in general n dimensions is constructed as the deformation of the abelian BF theory. The action of the abelian BF theory in n dimensions is defined as follows:

$$S_0 = \sum_{p=0}^{[\frac{n-1}{2}]} \int_X (-1)^{n-p} B_{n-p-1\ a_p} dA_p{}^{a_p}\,, \tag{3.1}$$

where $A_p{}^{a_p}$ is a p-form gauge field and $B_{n-p-1\ a_p}$ is a $(n-p-1)$-form auxiliary field. Indices a_p, b_p, c_p, etc. represent target space indices for the p-from $A_p{}^{a_p}$. Target spaces for different p-forms may be different. X is a base manifold on which the theory is defined. The sign factors $(-1)^{n-p}$ are introduced for convenience. This action has the following abelian gauge symmetry:

$$\begin{aligned}
\delta_0 A_p{}^{a_p} &= dc_{p-1}^{(p)a_p}\,, \\
\delta_0 B_{n-p-1\ a_p} &= dt_{n-p-2\ a_p}^{(n-p-1)}\,,
\end{aligned} \tag{3.2}$$

where $c_{p-1}^{(p)a_p}$ is a $(p-1)$-form gauge parameter and $t_{n-p-2\ a_p}^{(n-p-1)}$ is a $(n-p-2)$-form gauge parameter. (p) in $c_{p-1}^{(p)a_p}$ and $(n-p-1)$ in $t_{n-p-2\ a_p}^{(n-p-1)}$ represent that $c_{p-1}^{(p)a_p}$ is a gauge parameter for p-form $A_p{}^{a_p}$ and $t^{(n-p-1)}_{n-p-2\ a_p}$ is one for $(n-p-1)$-form $B_{n-p-1\ a_p}$, respectively. This gauge symmetry is reducible. Since $A_p{}^{a_p}$ is a p-form and $B_{n-p-1\ a_p}$ is a $n-p-1$-form, we need the following towers of the "ghost for ghosts" to analyze the complete gauge degrees of freedom:

$$\begin{aligned}
&\delta_0 A_p{}^{a_p} = dc^{(p)a_p}_{p-1}\,, \qquad \delta_0 B_{n-p-1\ a_p} = dt^{(n-p-1)}_{n-p-2\ a_p}\,,\\
&\delta_0 c^{(p)a_p}_{p-1} = dc^{(p)a_p}_{p-2}\,, \qquad \delta_0 t^{(n-p-1)}_{n-p-2\ a_p} = dt^{(n-p-1)}_{n-p-3\ a_p}\,,\\
&\vdots\\
&\delta_0 c^{(p)a_p}_{1} = dc^{(p)a_p}_{0}\,, \qquad \delta_0 t^{(n-p-1)}_{1\ a_p} = dt^{(n-p-1)}_{0\ a_p}\,,\\
&\delta_0 c^{(p)a_p}_{0} = 0\,, \qquad \delta_0 t^{(n-p-1)}_{0\ a_p} = 0\,,
\end{aligned} \tag{3.3}$$

where $c^{(p)a_p}_i$ are i-form gauge parameters and $t^{(n-p-1)}_{j\ a_p}$ are j-form gauge parameters. $i = 0,\ldots,p-1$ and $j=0,\ldots,n-p-2$.

We write the theory in the BV formalism. First we take $c^{(p)a_p}_i$ to be the FP ghost i-form with ghost number $p-i$, and $t^{(n-p-1)}_{j\ a_p}$ to be a j-form with the ghost number $n-p-1-j$. As usual, if the ghost number is odd/even, the fields are Grassmann odd/even.

For $A_p{}^{a_p}$, we introduce the antifield $A^{+(p)}_{n-p\ a_p}$, which is a $(n-p)$-form with the ghost number -1. For $B_{n-p-1\ a_p}$, $B^{+(n-p-1)a_p}_{p+1}$, which is a $(p+1)$-form with the ghost number -1. For $c^{(p)a_p}_i$, $c^{+(p)}_{n-i\ a_p}$, which is a $(n-i)$-form with the ghost number $-p-1+i$. For $t^{(n-p-1)}_{j\ a_p}$, $t^{+(n-p-1)a_p}_{n-j}$, which is a $(n-j)$-form with the ghost number $-n+p+j$.

For $A_p{}^{a_p}$ and $B_{n-p-1\ a_p}$, we define the corresponding superfields as

$$\begin{aligned}
\mathbf{A}_p{}^{a_p} &= c^{(p)a_p}_0 + c^{(p)a_p}_1 + \cdots + c^{(p)a_p}_{p-1} + A_p{}^{a_p} + B^{+(n-p-1)a_p}_{p+1} + \\
&\quad + t^{+(n-p-1)a_p}_{p+2} + \cdots + t^{+(n-p-1)a_p}_{n}\,,\\
\mathbf{B}_{n-p-1\ a_p} &= t^{(n-p-1)}_{0\ a_p} + t^{(n-p-1)}_{1\ a_p} + \cdots + t^{(n-p-1)}_{n-p-2\ a_p} + B_{n-p-1\ a_p} + A^{+(p)}_{n-p\ a_p} + \\
&\quad + c^{+(p)}_{n-p+1\ a_p} + \cdots + c^{+(p)}_{n\ a_p}\,.
\end{aligned} \tag{3.4}$$

The total degrees of $\mathbf{A}_p{}^{a_p}$ and $\mathbf{B}_{n-p-1\ a_p}$ are p and $n-p-1$, respectively. Since $\mathbf{A}_p^{a_p}$ and $\mathbf{B}_{n-p-1\ a_p}$ are the field-antifield pair, we can rewrite the antibracket on two superfields F and G from the definition of the antibracket (2.5) and superfields (3.4) as follows:

$$(\!(\, F\,,\, G\,)\!) \equiv \sum_{p=0}^{[\frac{n-1}{2}]} F\cdot \frac{\overleftarrow{\partial}}{\partial \mathbf{A}_p{}^{a_p}}\cdot\frac{\overrightarrow{\partial}}{\partial \mathbf{B}_{n-p-1\ a_p}}\cdot G - (-1)^{np} F\cdot \frac{\overleftarrow{\partial}}{\partial \mathbf{B}_{n-p-1\ a_p}}\cdot\frac{\overrightarrow{\partial}}{\partial \mathbf{A}_p{}^{a_p}}\cdot G\,. \tag{3.5}$$

$(\!(\,\cdot\,,\,\cdot\,)\!)$ is graded symmetric and satisfy the graded Leibniz rule and the graded Jacobi identity for the total degree of superfields.

The possible deformations of the BF theory in n dimensions, which have been obtained in the paper [3], are as follows:

$$S = S_0 + gS_1\,, \tag{3.6}$$

where

$$S_0 = \sum_{p=0}^{[\frac{n-1}{2}]} \int_X (-1)^{n-p} \mathbf{B}_{n-p-1\ a_p} \cdot d\mathbf{A}_p{}^{a_p} \,,$$

$$S_1 = \sum_{p(1),\ldots,p(k),q(1),\ldots,q(l)} \int_X F_{p(1)\cdots p(k),q(1)\cdots q(l)\ a_{p(1)}\cdots a_{p(k)}}{}^{b_{q(1)}\cdots b_{q(l)}}(\mathbf{A}_0{}^{a_0}) \times$$

$$\times \mathbf{A}_{p_1}{}^{a_{p(1)}} \cdots \mathbf{A}_{p_k}{}^{a_{p(k)}} \cdot \mathbf{B}_{q_1 b_{q(1)}} \cdots \mathbf{B}_{q_l b_{q(l)}} \,, \tag{3.7}$$

where $F_{p(1)\cdots p(k),q(1)\cdots q(l) a_{p(1)}\cdots a_{p(k)}}{}^{b_{q(1)}\cdots b_{q(l)}}(\mathbf{A}_0{}^{a_0})$ is a function of $\mathbf{A}_0{}^{a_0}$ and $p(r) \neq 0, q(s) \neq 0$ for $r = 1, \ldots, k, s = 1, \ldots, l$. The integration over X is understood to vanish except for the n-form part of the integrand. We require that the total degree of $\mathcal{L}_1$ is n, that is, the ghost number of the action is zero, as in the physical situation, though that is not necessarily required by mathematical consistency of the deformations.

A necessary and sufficient condition for the theory to be consistent is that the total action S satisfy the following classical master equation:

$$(\!(S \,,\, S)\!) = 0 \,. \tag{3.8}$$

It is easily confirmed that $\delta_0 S_0 = (\!(S_0 \,,\, S_0)\!) = 0$ and $\delta_0 S_1 = (\!(S_0 \,,\, S_1)\!) = 0$ if we take the proper boundary conditions in such a way that the integrals of total derivative terms vanish. Therefore the condition (2.10) reduces to

$$(\!(S_1 \,,\, S_1)\!) = 0 \,. \tag{3.9}$$

This condition imposes some identities on the structure functions $F_{p(1)\cdots p(k),q(1)\cdots q(l) a_{p(1)}\cdots a_{p(k)}}{}^{b_{q(1)}\cdots b_{q(l)}}(\mathbf{A}_0{}^{a_0})$ in (3.7).

The master eq. (3.8) reduces to the equation, $\delta_0 S_1 + g/2(\!(S_1 \,,\, S_1)\!) = 0$. This is nothing but the Maurer-Cartan equation with respect to the BRST differential δ_0. We have obtained a solution of "a flat equation" on the space of field theories.

The total BRST transformations δ for the superfields are calculated as

$$\delta \mathbf{A}_p{}^a = (-1)^{n-p} (\!(S \,,\, \mathbf{A}_p{}^a)\!) = d\mathbf{A}_p{}^{a_p} + (-1)^{n-p} \frac{\overrightarrow{\partial}}{\partial \mathbf{B}_{n-p-1\ a_p}} \cdot S_1 \,, \tag{3.10}$$

$$\delta \mathbf{B}_{n-p-1\ a_p} = (-1)^{p(n-p)} \big(\!\big(S \,,\, \mathbf{B}_{n-p-1\ a_p} \big)\!\big) = d\mathbf{B}_{n-p-1\ a_p} + (-1)^{p(n-p)} \frac{\overrightarrow{\partial}}{\partial \mathbf{A}_{p p}^{a}} \cdot S_1 \,.$$

4. Algebraic structures of deformed BF theories in lower dimensions

In this section, we analyze algebraic structures of deformed BF theory in lower dimensions.

4.1 In two dimensions

First, we analyze the algebraic structure of two-dimensional deformed BF theory as an example.

In two dimensions, (3.6) becomes

$$
\begin{aligned}
S &= S_0 + gS_1 \,, \\
S_0 &= \int_X \mathbf{B}_{1a} \cdot d\phi^a \,, \qquad S_1 = \int_\Sigma \frac{1}{2} f^{ab}(\phi^a) \cdot \mathbf{B}_{1a} \cdot \mathbf{B}_{1b} \,,
\end{aligned} \tag{4.1}
$$

where we rewrite the notations as $\phi^a = \mathbf{A}_0{}^a$ and $(1/2) f^{ab}(\phi^a) = F_{,11}(\mathbf{A}_0{}^a)$. From the condition (3.9), we obtain the following identity on f^{ab}:

$$
f^{cd} \cdot \frac{\overrightarrow{\partial}}{\partial \phi^d} \cdot f^{ab} + f^{ad} \cdot \frac{\overrightarrow{\partial}}{\partial \phi^d} \cdot f^{bc} + f^{bd} \cdot \frac{\overrightarrow{\partial}}{\partial \phi^d} \cdot f^{ca} = 0 \,. \tag{4.2}
$$

If we set all the antifields to zero, (4.2) is rewritten as

$$
f^{cd}(\phi) \frac{\partial f^{ab}(\phi)}{\partial \phi^d} + f^{ad}(\phi) \frac{\partial f^{bc}(\phi)}{\partial \phi^d} + f^{bd}(\phi) \frac{\partial f^{ca}(\phi)}{\partial \phi^d} = 0 \,. \tag{4.3}
$$

This theory is known as two-dimensional nonlinear gauge theory (the Poisson sigma model) [11, 12]. Under the identity (4.2), $-f^{ab}$ defines the Poisson structure as

$$
\{F(\phi), G(\phi)\} \equiv -f^{ab}(\phi) \frac{\partial F}{\partial \phi^a} \frac{\partial G}{\partial \phi^a} \,, \tag{4.4}
$$

on the target space M. Conversely, if we consider the Poisson structure $-f^{ab}$ on M, which satisfies the identity (4.2), we can define the action (4.1) consistently.

If we quantize this theory on a disc, correlation functions of observables on the boundary of the disc coincide with the deformation quantization on the Poisson manifold M [26]. We can derive the associativity of the deformation quantization from the gauge symmetry of the theory.

The gauge symmetry of this theory is considered not as a Lie algebra but as a Lie algebroid [17, 18]. A Lie algebroid is a generalization of bundles of Lie algebras over a base manifold $\mathcal{M}$. A Lie algebroid over a manifold is a vector bundle $\mathcal{E} \to \mathcal{M}$ with a Lie algebra structure on the space of the sections $\Gamma(\mathcal{E})$ defined by the bracket $[e_1, e_2]$, $e_1, e_2 \in \Gamma(\mathcal{E})$ and a bundle map (the anchor) $\rho : \mathcal{E} \to T\mathcal{M}$ satisfying the following properties:

$$
\begin{aligned}
\text{For any } e_1, e_2 \in \Gamma(\mathcal{E}), \qquad & [\rho(e_1), \rho(e_2)] = \rho([e_1, e_2]) \,, \\
\text{For any } e_1, e_2 \in \Gamma(\mathcal{E}),\ F \in C^\infty(\mathcal{M}) \,, \qquad & [e_1, F e_2] = F[e_1, e_2] + (\rho(e_1) F) e_2 \,,
\end{aligned} \tag{4.5}
$$

Here we construct the bracket of the Lie algebroid from the antibracket and the BRST structure of the theory. In our case, $\mathcal{M}$ is a space of smooth maps ϕ^a from the supermanifold $\tilde{X}$ whose degree zero part is a two-dimensional manifold X to a target space M. A vector bundle $\mathcal{E}$ is a cotangent bundle $T^*[1]\mathcal{M}$, where the grading of fiber direction is shifted by one. The Lie bracket of the two sections e_1 and e_2 is defined by

$$
[e_1, e_2] \equiv (\!(\, (\!(\, S \,,\, e_1 \,)\!) \,,\, e_2 \,)\!) \,, \tag{4.6}
$$

and the anchor is defined by

$$
\rho(e) F(\phi) \equiv (\!(\, e \,,\, (\!(\, S \,,\, F(\phi) \,)\!) \,)\!) \,. \tag{4.7}
$$

Then $[e_1, e_2] = -[e_2, e_1]$ is confirmed from the graded Jacobi identity of the antibracket and $(\!(e_1 \,,\, e_2)\!) = 0$. A Lie algebroid conditions (4.5) on the bracket $[\cdot,\cdot]$ and the anchor map ρ is obtained from the properties of the antibracket. The above definition defines the following "non-commutative" relation on the coordinates:

$$[\phi^a, \phi^b] = -f^{ab}(\phi)\,, \tag{4.8}$$

and the anchor is a differentiation on functions of ϕ as

$$\rho(\phi^a)F(\phi) = -f^{ab}(\phi) \cdot \frac{\overrightarrow{\partial}}{\partial \phi^b} \cdot F(\phi)\,. \tag{4.9}$$

4.2 In three dimensions

We consider the deformed BF theory in three dimensions. In this case, the theory defines the topological open 2-brane as a sigma model [19]. The total action (3.6) becomes as follows:

$$\begin{aligned}
S &= S_0 + gS_1\,, \\
S_0 &= \int_X [-\mathbf{B}_{2i} \cdot d\phi^i + \mathbf{B}_{1a} \cdot d\mathbf{A}_1{}^a]\,, \\
S_1 &= \int_X \Bigg[f_{1a}{}^i(\phi) \cdot \mathbf{A}_1{}^a \cdot \mathbf{B}_{2i} + f_2^{ib}(\phi) \cdot \mathbf{B}_{2i} \cdot \mathbf{B}_{1b} + \frac{1}{3!} f_{3abc}(\phi) \cdot \mathbf{A}_1{}^a \cdot \mathbf{A}_1{}^b \cdot \mathbf{A}_1{}^c + \\
&\qquad + \frac{1}{2} f_{4ab}{}^c(\phi) \cdot \mathbf{A}_1{}^a \cdot \mathbf{A}_1{}^b \cdot \mathbf{B}_{1c} + \frac{1}{2} f_{5a}{}^{bc}(\phi) \cdot \mathbf{A}_1{}^a \cdot \mathbf{B}_{1b} \cdot \mathbf{B}_{1c} + \\
&\qquad + \frac{1}{3!} f_6{}^{abc}(\phi) \cdot \mathbf{B}_{1a} \cdot \mathbf{B}_{1b} \cdot \mathbf{B}_{1c} \Bigg]\,,
\end{aligned} \tag{4.10}$$

where we set $f_{1a}{}^i = F_{1,2a}{}^i$, $f_2^{ib} = F_{,21}{}^{ib}$, $\frac{1}{3!} f_{3abc} = F_{111,abc}$, $\frac{1}{2} f_{4ab}{}^c = F_{11,1ab}{}^c$, $\frac{1}{2} f_{5a}{}^{bc} = F_{1,11a}{}^{bc}$, $\frac{1}{3!} f_6{}^{abc} = F_{,111}{}^{abc}$, for clarity. The condition of the classical master eq. (3.9) imposes the following identities on six f_i's, $i = 1, \ldots, 6$ [2]:

$$\begin{aligned}
f_{1e}{}^i \cdot f_2{}^{je} + f_2{}^{ie} \cdot f_{1e}{}^j &= 0\,, \\
\left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{1c}{}^i\right) \cdot f_{1b}{}^j - \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{1b}{}^i\right) \cdot f_{1c}{}^j + f_{1e}{}^i \cdot f_{4bc}{}^e + f_2{}^{ie} \cdot f_{3ebc} &= 0\,, \\
-f_{1b}{}^j \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_2{}^{ic}\right) + f_2{}^{jc} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{1b}{}^i\right) + f_{1e}{}^i \cdot f_{5b}{}^{ec} - f_2{}^{ie} \cdot f_{4eb}{}^c &= 0\,, \\
f_2{}^{jb} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_2{}^{ic}\right) - f_2{}^{jc} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_2{}^{ib}\right) + f_{1e}{}^i \cdot f_6^{ebc} + f_2{}^{ie} \cdot f_{5e}{}^{bc} &= 0\,, \\
f_{1[a}{}^j \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{4bc]}{}^d\right) - f_2{}^{jd} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{3abc}\right) + f_{4e[a}{}^d \cdot f_{4bc]}{}^e + f_{3e[ab} \cdot f_{5c]}{}^{de} &= 0\,, \\
f_{1[a}{}^j \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{5b]}{}^{cd}\right) + f_2{}^{j[c} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{4ab}{}^{d]}\right) + f_{3eab} \cdot f_6{}^{ecd} + &
\end{aligned}$$

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$$
\begin{aligned}
&+ f_{4e[a}{}^{[d} \cdot f_{5b]}{}^{c]e} + f_{4ab}{}^{e} \cdot f_{5e}{}^{cd} = 0\,, \\
f_{1a}{}^{j} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_6{}^{bcd}\right) - f_2{}^{j[b} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{5a}{}^{cd]}\right) &+ f_{4ea}{}^{[b} \cdot f_6{}^{cd]e} + f_{5e}{}^{[bc} \cdot f_{5a}{}^{d]e} = 0\,, \\
f_2{}^{j[a} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_6{}^{bcd]}\right) &+ f_6{}^{e[ab} \cdot f_{5e}{}^{cd]} = 0\,, \\
f_{1[a}{}^{j} \cdot \left(\frac{\overrightarrow{\partial}}{\partial \phi^j} \cdot f_{3bcd]}\right) &+ f_{4[ab}{}^{e} \cdot f_{3cd]e} = 0\,, \qquad (4.11)
\end{aligned}
$$

where $[\cdots]$ on the indices represents the antisymmetrization of them, e.g. $\Phi_{[ab]} = \Phi_{ab} - \Phi_{ba}$.

If we set all the antifields to zero, we obtain the ordinary action without antifields as follows:

$$
\begin{aligned}
S &= S_0 + g S_1\,, \\
S_0 &= \int_X \left[-B_{2\,i} d\phi^i + B_{1\,a} dA_1{}^a \right], \\
S_1 &= \int_X \Big[f_{1a}{}^{i}(\phi) A_1{}^a B_{2\,i} + f_2{}^{ib}(\phi) B_{2i} B_{1b} + \frac{1}{3!} f_{3abc}(\phi) A_1{}^a A_1{}^b A_1{}^c + \\
&\quad + \frac{1}{2} f_{4ab}{}^{c}(\phi) A_1{}^a A_1{}^b B_{1c} + \frac{1}{2} f_{5a}{}^{bc}(\phi) A_1{}^a B_{1b} B_{1c} + \frac{1}{3!} f_6{}^{abc}(\phi) B_{1a} B_{1b} B_{1c} \Big]. \qquad (4.12)
\end{aligned}
$$

And (4.11) reduces to the following identities:

$$
\begin{aligned}
f_{1e}{}^{i} f_2{}^{je} + f_2{}^{ie} f_{1e}{}^{j} &= 0\,, \\
\frac{\partial f_{1c}{}^{i}}{\partial \phi^j} f_{1b}{}^{j} - \frac{\partial f_{1b}{}^{i}}{\partial \phi^j} f_{1c}{}^{j} + f_{1e}{}^{i} f_{4bc}{}^{e} + f_2{}^{ie} f_{3ebc} &= 0\,, \\
- f_{1b}{}^{j} \frac{\partial f_2{}^{ic}}{\partial \phi^j} + f_2{}^{jc} \frac{\partial f_{1b}{}^{i}}{\partial \phi^j} + f_{1e}{}^{i} f_{5b}{}^{ec} - f_2{}^{ie} f_{4eb}{}^{c} &= 0\,, \\
f_2{}^{jb} \frac{\partial f_2{}^{ic}}{\partial \phi^j} - f_2{}^{jc} \frac{\partial f_2{}^{ib}}{\partial \phi^j} + f_{1e}{}^{i} f_6^{ebc} + f_2{}^{ie} f_{5e}{}^{bc} &= 0\,, \\
f_{1[a}{}^{j} \frac{\partial f_{4bc]}{}^{d}}{\partial \phi^j} - f_2{}^{jd} \frac{\partial f_{3abc}}{\partial \phi^j} + f_{4e[a}{}^{d} f_{4bc]}{}^{e} + f_{3e[ab} f_{5c]}{}^{de} &= 0\,, \\
f_{1[a}{}^{j} \frac{\partial f_{5b]}{}^{cd}}{\partial \phi^j} + f_2{}^{j[c} \frac{\partial f_{4ab}{}^{d]}}{\partial \phi^j} + f_{3eab} f_6{}^{ecd} + f_{4e[a}{}^{[d} f_{5b]}{}^{c]e} + f_{4ab}{}^{e} f_{5e}{}^{cd} &= 0\,, \\
f_{1a}{}^{j} \frac{\partial f_6{}^{bcd}}{\partial \phi^j} - f_2{}^{j[b} \frac{\partial f_{5a}{}^{cd]}}{\partial \phi^j} + f_{4ea}{}^{[b} f_6{}^{cd]e} + f_{5e}{}^{[bc} f_{5a}{}^{d]e} &= 0\,, \\
f_2{}^{j[a} \frac{\partial f_6{}^{bcd]}}{\partial \phi^j} + f_6{}^{e[ab} f_{5e}{}^{cd]} &= 0\,, \\
f_{1[a}{}^{j} \frac{\partial f_{3bcd]}}{\partial \phi^j} + f_{4[ab}{}^{e} f_{3cd]e} &= 0\,. \qquad (4.13)
\end{aligned}
$$

We consider the supermanifold $\tilde{X}$ whose grade zero part is a three-dimensional manifold X. In our topological field theory, a base space $\mathcal{M}$ is the space of a (smooth) map from $\tilde{X}$ to a target space M. We consider the same setting in section 2. The parity of the fiber of $\mathcal{E}$ is reversed and the grading is shifted by one. The fiber is $\mathcal{V}[1] \oplus \mathcal{V}^*[1]$, where $\mathcal{V}$ is a vector space and $[p]$ represents the grading shifted by p. We introduce a graded odd Poisson bracket (the antibracket) $(\!(\cdot\,,\,\cdot)\!)$ on the space. We take a local basis on $\Gamma(\mathcal{E})$ as $e^a = \mathbf{A}_1{}^a, \mathbf{B}_{1\ a}$, which are Darboux coordinates such that $\left(\!\left(\mathbf{A}_1{}^a\,,\, \mathbf{A}_1{}^b \right)\!\right) = (\!(\mathbf{B}_{1a}\,,\, \mathbf{B}_{1b})\!) = 0$ and $(\!(\mathbf{A}_1{}^a\,,\, \mathbf{B}_{1b})\!) = \delta^a{}_b$. In other words, $\mathbf{A}_1{}^a$ and $\mathbf{B}_{1\ a}$ are BV field-antifield pairs.

We define a graded symmetric bilinear form $\langle\cdot,\cdot\rangle$, a bilinear operation $\circ$ and a bundle map ρ from the antibracket as follows:

$$
\begin{aligned}
e^a \circ e^b &\equiv \left(\!\left((\!(S\,,\, e^a)\!)\,,\, e^b \right)\!\right), \\
\langle e^a, e^b\rangle &\equiv \left(\!\left(e^a\,,\, e^b \right)\!\right), \\
\rho(e^a)F(\phi) &\equiv (\!(e^a\,,\, (\!(S\,,\, F(\phi))\!))\!)\,, \\
\mathcal{D}(*) &\equiv (\!(S\,,\, *)\!)\,.
\end{aligned} \tag{4.14}
$$

Then we can confirm that the gauge algebra satisfies the conditions (2.13) to (2.17) of the Courant algebroid defined in section 2 from the identity (4.11) on structure functions f's. This theory is also considered as a generalization of the model in section 2.

Conversely, first we define the graded odd Poisson structure $\langle\cdot,\cdot\rangle$ on the bundle $\mathcal{E} \oplus T^*[2]\mathcal{M}$, where the grading on the fiber direction of $T^*[2]\mathcal{M}$ is shifted by 2. We define the operations $\circ$ and ρ on the basis as follows:

$$
\begin{aligned}
\mathbf{A}_1{}^a \circ \mathbf{A}_1{}^b &= -f_{5c}{}^{ab}(\phi)\cdot \mathbf{A}_1{}^c - f_6{}^{abc}(\phi)\cdot \mathbf{B}_{1c}\,, \\
\mathbf{A}_1{}^a \circ \mathbf{B}_{1b} &= -f_{4bc}{}^{a}(\phi)\cdot \mathbf{A}_1{}^c + f_{5b}{}^{ac}(\phi)\cdot \mathbf{B}_{1c}\,, \\
\mathbf{B}_{1a} \circ \mathbf{B}_{1b} &= -f_{3abc}(\phi)\cdot \mathbf{A}_1{}^c - f_{4ab}{}^{c}(\phi)\cdot \mathbf{B}_{1c}\,, \\
\rho(\mathbf{A}_1{}^a)\phi^i &= -f_2{}^{ia}(\phi)\,, \\
\rho(\mathbf{B}_{1a})\phi^i &= -f_{1a}{}^{i}(\phi)\,.
\end{aligned} \tag{4.15}
$$

Then the conditions (2.13) to (2.17) of the Courant algebroid are equivalent to the identities (4.11) on six f's. Since the master eq. (2.10) is equivalent to (4.11), the relations (2.13) to (2.17) is equivalent to the master equation of the action S.

The theory define the Courant algebroid structure on the target space as a sigma model. Therefore we can consider this model as "the Courant sigma model". We find that the topological open 2-brane has the Courant algebroid structure.

5. Algebraic structure in n dimensions

We discuss the structures of n-dimensional deformed BF theories in this section. The Batalin-Vilkovisky (antifield BRST) formalism is the key device to analyze general deformed BF theories.

A graded supermanifold M with non-negative integer grading is an N-manifold if the integer grading is compatible with parity. This means that bosonic fields have even weights and fermionic fields have odd weights [25, 27]. Our superfields satisfy this condition. A P-manifold is defined as a supermanifold equipped with an odd non-degenerate closed 2-form. This 2-form defines the odd Poisson bracket, which is nothing but the antibracket.

A Q-manifold is a supermanifold endowed with a degree $+1$ vector field Q whose square is zero, $Q^2 = 0$. Q is nothing but the BRST charge and realized by the BV action. A QP-manifold is defined as a Q-manifold with an odd symplectic structure which is Q-invariant. The antibracket and the BV action as the solution of the master equation define a QP-structure [21]. The sigma models based on our topological field theories have the NPQ-structures, and induce the geometry with the NPQ-structures on the target space.

Let X be a n-dimensional manifold and let $\tilde{X}$ be a supermanifold whose grade zero part is a n-dimensional manifold X. A base space $\mathcal{M}$ is the space of a (smooth) map $\phi^{a_0} = \mathbf{A}_0{}^{a_0}$ from $\tilde{X}$ to a target space M. The fiber of $\mathcal{E}$ is graded. The fiber is $\oplus_{p=1}^{[(n-1)/2]}(\mathcal{V}_p[p] \oplus \mathcal{V}_p^*[n-p-1])$, where $\mathcal{V}_p$, $p = 1, \ldots, [(n-1)/2]$ are vector spaces and $[p]$ in $\mathcal{V}_p[p]$ represents the grading shifted by p. $[n-p-1]$ in $\mathcal{V}_p^*[n-p-1]$ represents the grading shift by $n-p-1$. Grading is 1 to $n-2$ and compatible with the parity.

The Σ_n-manifold is an NQ-manifold with a Q-invariant symplectic form of degree n [27]. In our model, the Σ_{n-1}-structure is realized by the n-dimensional topological field theory.

We take a local basis on $\Gamma(\mathcal{E})$ as $e^a = \mathbf{A}_p{}^{a_p}, \mathbf{B}_{n-p-1\ a_p}$, where $p \neq 0$. We introduce a graded odd Poisson bracket (the antibracket) $(\!(\,\cdot\,,\,\cdot\,)\!)$ on the space $\mathcal{E} \oplus T^*[n-1]\mathcal{M}$, where the grading on the fiber direction of $T^*[n-1]\mathcal{M}$ is shifted by $n-1$. $\mathbf{A}_p{}^{a_p}$ and $\mathbf{B}_{n-p-1\ a_p}$ are Darboux coordinates such that

$$\begin{aligned} \Big(\!\Big(\, \mathbf{A}_p{}^{a_p} \,,\, \mathbf{A}_q{}^{b_q} \,\Big)\!\Big) &= (\!(\, \mathbf{B}_{n-p-1\ a_p} \,,\, \mathbf{B}_{n-q-1\ b_q} \,)\!) = 0 \,, \\ (\!(\, \mathbf{A}_p{}^{a_p} \,,\, \mathbf{B}_{n-q-1\ b_q} \,)\!) &= \delta^p{}_q \delta^{a_p}{}_{b_q} \,. \end{aligned} \tag{5.1}$$

We define the following three operations:

$$\langle E_1, E_2 \rangle \equiv (\!(\, E_1 \,,\, E_2 \,)\!) \,, \tag{5.2}$$

$$\tau(E_1 \,, E_2) \equiv (\!(\, (\!(\, S \,,\, E_1 \,)\!) \,,\, E_2 \,)\!) \,, \tag{5.3}$$

$$\mathcal{D}(*) \equiv (\!(\, S \,,\, * \,)\!) \,, \tag{5.4}$$

where $E_1, E_2 \in \Gamma(\mathcal{E})$ or $\in C^\infty(\mathcal{M})$. Note that the degree of $\langle \cdot, \cdot \rangle$ is $-n+1$, the degree of $\tau(\,\cdot\,, \cdot)$ is $-n+2$, and the degree of $\mathcal{D}$ is 1. $\mathcal{D}$ is a differentiation. We can generalize three operations on functions of $\mathbf{A}_p{}^{a_p}$, $p = 0, \ldots, [(n-1)/2]$, and $\mathbf{B}_{n-q-1\ a_q}$, $p = 1, \ldots, [(n-1)/2]$. $\langle E_1, E_2 \rangle$ is a graded symmetric bilinear form from the property of the antibracket as follows:

$$\langle E_1, E_2 \rangle = -(-1)^{(|E_1|+1-n)(|E_2|+1-n)} \langle E_2, E_1 \rangle \,. \tag{5.5}$$

Three operations are not independent. In fact, the following identity is satisfied:

$$\langle \mathcal{D}E_1, E_2 \rangle = \tau(E_1 \,, E_2) \,, \tag{5.6}$$

because both sides are equal to $(\!(\, (\!(\, S \,,\, E_1 \,)\!) \,,\, E_2 \,)\!)$. We can prove the following identities including the graded symmetric property of $\langle \cdot, \cdot \rangle$, derivation properties and the Jacobi identities for $\langle \cdot, \cdot \rangle$ and $\tau(\,\cdot\,, \cdot)$, from the properties of the antibracket and the BV action S

of the deformed BF theories:

$$\langle E_1, E_2\rangle = -(-1)^{(|E_1|+1-n)(|E_2|+1-n)}\langle E_2, E_1\rangle, \tag{5.7}$$

$$\langle E_1, E_2\cdot E_3\rangle = \langle E_1, E_2\rangle\cdot E_3 + (-1)^{(|E_1|+1-n)|E_2|}E_2\cdot\langle E_1, E_3\rangle, \tag{5.8}$$

$$(-1)^{(|E_1|+1-n)(|E_3|+1-n)}\langle E_1, \langle E_2, E_3\rangle\rangle + \text{cyclic permutations} = 0\,, \tag{5.9}$$

$$\tau(E_1\,, E_2\cdot E_3) = \tau(E_1\,, E_2)\cdot E_3 + (-1)^{(|E_1|-n)|E_2|}E_2\cdot\tau(E_1\,, E_3)\,, \tag{5.10}$$

$$\tau(E_1\,, \langle E_2, E_3\rangle) = \langle\tau(E_1\,, E_2), E_3\rangle + (-1)^{(|E_1|-n)(|E_2|+1-n)}\langle E_2, \tau(E_1\,, E_3)\rangle, \tag{5.11}$$

$$\begin{aligned}\tau(E_1\,, \tau(E_2\,, E_3)) = {}& (-1)^{(|E_1|-n)}\tau(\tau(E_1\,, E_2)\,, E_3) + \\ & + (-1)^{(|E_1|-n)(|E_2|-n)}\tau(E_2\,, \tau(E_1\,, E_3))\,.\end{aligned} \tag{5.12}$$

Generally, $\tau(E_1\,, E_2)$ is not graded antisymmetric as

$$\tau(E_1\,, E_2) = (-1)^{(|E_1|+1-n)(|E_2|+1-n)}\tau(E_2\,, E_1) + \mathcal{D}\langle E_1, E_2\rangle\,. \tag{5.13}$$

If the last term $\mathcal{D}\langle E_1, E_2\rangle$ vanish, $\tau(\cdot\,,\cdot)$ is graded antisymmetric. We can define a graded antisymmetric bracket $[E_1, E_2]$ as follows:

$$\tau(E_1\,, E_2) = [E_1, E_2] + \frac{1}{2}\mathcal{D}\langle E_1, E_2\rangle\,, \tag{5.14}$$

or

$$[E_1, E_2] \equiv \frac{1}{2}\left\{\tau(E_1\,, E_2) + (-1)^{(|E_1|+1-n)(|E_2|+1-n)}\tau(E_1\,, E_2)\right\}. \tag{5.15}$$

In fact, $[\cdot,\cdot]$ is graded antisymmetric as

$$[E_2, E_1] = (-1)^{(|E_1|+1-n)(|E_2|+1-n)}[E_1, E_2]\,. \tag{5.16}$$

However, in the theory in dimensions higher than two, this graded antisymmetric bracket does not generally satisfy the graded Jacobi identity. It is understood as a breaking of the Jacobi identity on the Courant algebroid in three dimensions. The operations which satisfy the Jacobi identities are $\tau(\cdot\,,\cdot)$ and $\langle\cdot,\cdot\rangle$.

In the deformed BF theory in two dimensions, E is a function of ϕ^a. We can confirm that the algebra is the same one in the section 4.1. We find $\langle E_1, E_2\rangle = 0$ and $\tau(E_1\,, E_2) = [E_1, E_2]$. The anchor is essentially the same with $\tau(\cdot\,,\cdot)$, since $[E_1, E_2] = -\rho(E_2)E_1$.

In three-dimensional deformed BF theory, E is expanded by ϕ^i, $\mathbf{A}_1{}^a$ and $\mathbf{B}_{1a}$. $\langle E_1, E_2\rangle = 0$ if E_1 or E_2 is a function of ϕ^i. If E_1 and E_2 are sections of $\mathcal{E}$, $\langle E_1, E_2\rangle$ is equal to $\langle\cdot,\cdot\rangle$ in section 4.2. The τ operation satisfies that $\tau(F(\phi)\,, G(\phi)) = 0$, $\tau(F(\phi)\,, e) = \rho(e)F(\phi)$ and $\tau(e_1\,, e_2) = e_1\circ e_2$, where e, e_1 and e_2 are sections of $\mathcal{E}$. If we substitute the relations in the above, we can confirm the five conditions (2.13)–(2.17) of the Courant algebroid from the identities (5.10)–(5.13). The condition (2.13) of the Courant algebroid is the graded Jacobi identity (5.12). If one of E is a function of ϕ^i in (5.12), we obtain the condition (2.14). The condition (2.15) follows from (5.10) if E_2 is a function of ϕ^i. The condition (2.16) is obtained from (5.13), the condition (2.17) from (5.11).

Conversely, we assume $\langle E_1, E_2\rangle$, $\tau(\cdot\,,\cdot)$ and $\mathcal{D}$ on $\Gamma(\mathcal{E})$ which satisfy (5.6) to (5.12) on an N-manifold $\mathcal{E}\oplus T^*[n-1]\mathcal{M}$, where the degree of $\langle\cdot,\cdot\rangle$ is $-n+1$, the degree of $\tau(\cdot\,,\cdot)$ is $-n+2$, and the degree of $\mathcal{D}$ is 1. If at least one of E is a function of ϕ, $\tau(E_1\,, E_2)$ is considered as the anchor. $\tau(F(\phi)\,, G(\phi)) = 0$ if $n \geq 3$ because we consider N-manifold with "non-negative" integer degree and the degree, $-n+2$, of the left hand side is negative. Then we can prove $\mathcal{D}^2 = 0$ is equivalent with the compatibility of (5.6), the Jacobi identities (5.12) and (5.9). Therefore the algebra defines the QP-structure on the target space. The Q-structure is $\langle E_1, E_2\rangle$ and the P-structure is $\mathcal{D}$.

$\langle\cdot,\cdot\rangle$ is the graded odd Poisson bracket on $\mathcal{E}\oplus T^*[n-1]\mathcal{M}$, therefore we can find the hamiltonian S for the vector field $\mathcal{D}$ such that $\mathcal{D}E = \langle S, E\rangle$. The solution is our BV action. $\mathcal{D}^2 = 0$ is equivalent to the classical master equation $\langle S, S\rangle = 0$.

The algebroid with $\langle E_1, E_2\rangle$, $\tau(\cdot\,,\cdot)$ and $\mathcal{D}$ which satisfies the identities (5.6) to (5.12) is the Batalin-Vilkovisky algebra based on the topological field theory in n dimensions. The deformed BF theory in n dimensions defines the above algebroid structures on the target space as "a Batalin-Vilkovisky sigma model".

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A. Antibracket

In n dimensions, we define the antibracket for functions $F(\Phi,\Phi^+)$ and $G(\Phi,\Phi^+)$ of the fields and the antifields as follows:

$$(F,G) \equiv \frac{F\overleftarrow{\partial}}{\partial\Phi}\frac{\overrightarrow{\partial}G}{\partial\Phi^+} - \frac{F\overleftarrow{\partial}}{\partial\Phi^+}\frac{\overrightarrow{\partial}G}{\partial\Phi}\,, \tag{A.1}$$

where $\overleftarrow{\partial}/\partial\varphi$ and $\overrightarrow{\partial}/\partial\varphi$ are the right differentiation and the left differentiation with respect to φ, respectively. The following identity about the left and right derivative is useful,

$$\frac{\overrightarrow{\partial}F}{\partial\varphi} = (-1)^{(\mathrm{gh}F-\mathrm{gh}\varphi)\mathrm{gh}\varphi+(\deg F-\deg\varphi)\deg\varphi}\frac{F\overleftarrow{\partial}}{\partial\varphi}\,. \tag{A.2}$$

For two functionals S and T, the antibracket is defined as follows:

$$(S,T) \equiv \int_X \left(\frac{S\overleftarrow{\partial}}{\partial\Phi}\frac{\overrightarrow{\partial}T}{\partial\Phi^+} - \frac{S\overleftarrow{\partial}}{\partial\Phi^+}\frac{\overrightarrow{\partial}T}{\partial\Phi}\right). \tag{A.3}$$

The antibracket satisfies the following identities:

$$\begin{aligned}
(F,G) &= -(-1)^{(\deg F-n)(\deg G-n)+(\mathrm{gh}F+1)(\mathrm{gh}G+1)}(G,F)\,,\\
(F,GH) &= (F,G)H + (-1)^{(\deg F-n)\deg G+(\mathrm{gh}F+1)\mathrm{gh}G}G(F,H)\,,\\
(FG,H) &= F(G,H) + (-1)^{\deg G(\deg H-n)+\mathrm{gh}G(\mathrm{gh}H+1)}(F,H)G\,,\\
(-1)^{(\deg F-n)(\deg H-n)+(\mathrm{gh}F+1)(\mathrm{gh}H+1)}&(F,0(G,H)) + \text{cyclic permutations} = 0\,,
\end{aligned} \tag{A.4}$$

in n dimensions, where F, G and H are functions on fields and antifields.

B. Dot product

For superfields $F(\Phi, \Phi^+)$ and $G(\Phi, \Phi^+)$, the following identities are satisfied:

$$\begin{aligned} FG &= (-1)^{\mathrm{gh}F\mathrm{gh}G+\deg F \deg G} GF \,, \\ d(FG) &= dFG + (-1)^{\deg F} F dG \,, \end{aligned} \tag{B.1}$$

as the usual products. The graded commutator of two superfields satisfies the following identities:

$$\begin{aligned} [F, G] &= -(-1)^{\mathrm{gh}F\mathrm{gh}G+\deg F \deg G} [G, F] \,, \\ [F, [G, H]] &= [[F, G], H] + (-1)^{\mathrm{gh}F\mathrm{gh}G+\deg F \deg G} [G, [F, H]] \,. \end{aligned} \tag{B.2}$$

We introduce the total degree of a superfield F as $|F| = \mathrm{gh}F + \deg F$. We define the *dot product* on superfields as

$$F \cdot G \equiv (-1)^{\mathrm{gh}F \deg G} FG \,, \tag{B.3}$$

and the *dot Lie bracket*

$$[\![F\,, G]\!] \equiv (-1)^{\mathrm{gh}F \deg G} [F, G] \,. \tag{B.4}$$

We obtain the following identities of the *dot product* and the *dot Lie bracket* from (B.1), (B.2), (B.3), and (B.4),

$$\begin{aligned} F \cdot G &= (-1)^{|F||G|} G \cdot F \,, \\ [\![F\,, G]\!] &= -(-1)^{|F||G|} [\![G\,, F]\!] \,, \\ [\![F\,, [\![G\,, H]\!]]\!] &= [\![[\![F\,, G]\!]\,, H]\!] + (-1)^{|F||G|} [\![G\,, [\![F\,, H]\!]]\!] \,, \end{aligned} \tag{B.5}$$

and

$$d(F \cdot G) \equiv dF \cdot G + (-1)^{|F|} F \cdot dG \,. \tag{B.6}$$

The *dot antibracket* of the superfields F and G is defined as

$$(\!(\, F \,, \, G \,)\!) \equiv (-1)^{(\mathrm{gh}F+1)(\deg G - n)} (-1)^{\mathrm{gh}\Phi(\deg \Phi - n)+n} (F, G) \,. \tag{B.7}$$

Then the following identities are obtained from the eqs. (A.4) and (B.7):

$$\begin{aligned} (\!(\, F \,, \, G \,)\!) &= -(-1)^{(|F|+1-n)(|G|+1-n)} (\!(\, G \,, \, F \,)\!) \,, \\ (\!(\, F \,, \, G \cdot H \,)\!) &= (\!(\, F \,, \, G \,)\!) \cdot H + (-1)^{(|F|+1-n)|G|} G \cdot (\!(\, F \,, \, H \,)\!) \,, \\ (\!(\, F \cdot G \,, \, H \,)\!) &= F \cdot (\!(\, G \,, \, H \,)\!) + (-1)^{|G|(|H|+1-n)} (\!(\, F \,, \, H \,)\!) \cdot G \,, \\ (-1)^{(|F|+1-n)(|H|+1-n)} (\!(\, F \,, \, (\!(\, G \,, \, H \,)\!) \,)\!) &+ \text{cyclic permutations} = 0 \,. \end{aligned} \tag{B.8}$$

We define the *dot differential* as

$$\begin{aligned} \frac{\overrightarrow{\partial}}{\partial \varphi} \cdot F &\equiv (-1)^{\mathrm{gh}\varphi \deg F} \frac{\overrightarrow{\partial} F}{\partial \varphi} \,, \\ F \cdot \frac{\overleftarrow{\partial}}{\partial \varphi} &\equiv (-1)^{\mathrm{gh}F \deg \varphi} \frac{F \overleftarrow{\partial}}{\partial \varphi} \,. \end{aligned} \tag{B.9}$$

Then, from the eq. (A.2), we can obtain the formula

$$\frac{\overrightarrow{\partial}}{\partial\varphi} \cdot F = (-1)^{(|F|-|\varphi|)|\varphi|} F \cdot \frac{\overleftarrow{\partial}}{\partial\varphi}. \tag{B.10}$$

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